AE3030 Notes up till page 75

I'm going to type up everything I know about Aerodynamics SO FAR
It is September 10th today. The first quiz is on September 22nd

##

PROPERTIES OF FLUIDS

definite mass but indefinite volume
viscosity
--solids undergo static deformation
--fluid is a substance that deforms continuously under the action of a shearingforce
--if there is zero shear stress, then we are in **hydrostatic stress codition**

CONTINUUM
-avg behavior of particles/molecules
--1 mole of O2 is 6.02*10^23 O2's

-Mean free path
The avg distance a particle travels before colliding with another

Knudsen number
-This is the mean free path divided by a characteristic length scale, such as chord length or diameter or airfoil length

When the Knudsen number is small, the continuum model is of value
Kn number is represented by lambda
If the Knudsen number is 0.1 and under, we can assume continuum

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Some dimensional analysis
1 slug is 1 lbf s^2 / ft = 1 lbm(32.2 ft/s^2
1 lbf = 1 lbm * 32.2 ft/s^2

FLUID UNITS

Density

is mass over volume
m/ l^3

kg/ m^3

slug/ft^3

Incompressible flows when changes in density can neglected -- no fluid is ever actually incompressible

-formal definition

density = the limit as the change in volume approaches zero of the change in mass over the change in volume

Density is a thermodynamic property, meaning it relates to Temperature AND pressure

Density is also equal to P/RT (the R is the specific gas constant

PV = nRT (the universal gas law

##

Pressure

Pressure is the rate of change of of momentum (?)

Pressure = Force / Area = Pa = N/m^2 = lbf/ft^2

Make sure you know that P - P_inf is gauge pressure, which is comparing absolute pressure against a reference pressure. There is no such thing as a negative absolute pressure.

##

Temperature
A particle by itself has 3 degrees of translational motion and a group of particles (a duo of particles) has 2 degrees of rotational motion (the rotiation about its axis is ignored because it's so small)

Altogether that makes 5 degrees of freedom

Calorically perfect gas
: cp and cv are constant. cv = 5/2 * R
A thermally perfect gas is when cv and cp are functions of temperature

Real gas is when cv and cp are fxns of temperature and pressure

UNITS

K and R
K is C + 273.15

FLOW VELOCITY

-u,v,w: bulk velocities / average velocity

CONSERVATION EQUATIONS
Definition of a streamline
-a curve whose tangent at any point is in the direction of the velocity vector at that point
t = constant

streamlines in 3D are stream TUBES

Unsteady Flow - when the streamlines change

In order to use bernoulli's eq, you need to make sure the flow is steady, so make sure to pick an appropriate reference frame!

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Conservation of Mass/ Continuity Equations

The integral with respect to time over a control volume of density times the volume differential (unsteady term) plus the line integral over a control surface of the density multiplied with the velocity dotted with a normal vector multiplied by that control area (flux term) differential is all equal to zero.

d/dt*int_cv (rho*dV) + line-int_cs(rho*(v*n)*dA) = 0

When you assume steady flow, you can ignore the first term.

-rho1*V1*A1 i is the mass flow rate in

rho2v2A2 is the mass flow rate out

If you assume compressible, the densities cancel
V1A1 = V2A2

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Conservation of Momentum Equation

Rate of change of momentum is equal to the mass flow rate times velocity

(derivations on page 6 and 7)

Eurler's equation:

rho*V*dV + dP = 0

(assumtions: steady, inviscid, no body forces)
Bernoulli's equation:

Po = P + rho*v^2 / 2assumptions: incompressible, steady, inviscid, no body forces

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Thermodynamics 1st law

delta U = Q - W

Internal energy change is equal to the het added to the system minus the work done by the system
U = cv*T

Other forms of energy
-Kinetic energy
-Potential energy (gravitational, electric potential) -- largely ignored

-Assumptions: steady, no body forces, inviscid flow, adiabatic

Work done - viscous force (ignore)
pressure flow work
-shaft work (ifnore)

Rate at which work is being done is called Power
Power = F*velocity

(derivation on page 9-10)

Energy equation derivation continues on page 12

P = rho*R*T is called an equation of state

Enthalpy and relations between R, Cv, Cp, and gamma (maybe)

Stagnation enthalpy

h0 = h + 1/2v^2

Static enthalpy is equal to cp*T

cp is equal to cv + R

Summary of Energy

for a stream tube:
-rho*A*V is constant
pVdV + dP = 0 => Euler's equation
h + v^2/2 = h0 = constant
ASSUMING: steady, inviscid, no body forces, adiabatic

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2nd LAW OF THERMODYNAMICS

ds >= δq / T ds = δq/T + ds ← and ds >=0, so everything is irreversible

The total entropy of a system and surroundings is always increasing or at best stays the same

entropy => state variable (path independent)
heat added => path dependent

ds you can integrate
δq you cannot integrate, hence the weid lower-case delta

Define a Reversible process:

-one in which no dissipative phenomenon occurs
i.e., viscosity, thermal conductivity & mass diffiyusion are absent
-isentropic fokw => adiabatic and reversible

Non-isentropic
=>shock so hat Po decreases
=> boundary layers / viscous fluids

One form of the 2nd law:

$s2 – s1 = c_p ln (T2/T1) – R ln(P2/P1)$

Isentropic means that s2-s1 = 0

you can derive a temperature – pressure relation for isentropic flow (derivation on page 14

Also, ther eis another version of the 2nd law

$s2-s1 = c_v ln(T2/T1) + R ln(v2/v1)$ where v is specific volume in this case

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INTRO TO AIRFOILS (page 15)

uses of airfoils:
-wings, props and fans, helicopters, compressors and turbines, hydrofoils, wind turbines, etc

Evolution of airfoils.

in 1873 they were like a line,… had a sharp leading edge, in 1903 there was a thicker leading edge but the whole thing was still thin – biplane

by 1917 we got the modern airfoil

AIRFOIL VOCAB

Chordline
-connects leading edge and trailing edge
camber
-is in terms of the chordline
thickness
– perpedicular to camber line, midpoint is on camber line

Symmetric airfoil
- ➔ camber collapses to the chordline

angle of attack – angle between freestream and the chordline

Units for angle of attack
- ➔ always use radians!!

alpha is not constant across a wing – wings are twisted --- rotor blades, too, but even moreso
The root of the wing typically has a higher alpha and the tip has lower, and maybe even negative alpha

FORCES

Lift
-force acting perpendicular to V_inf
Drag
-Force acting along as V_inf
Moment b/c the l and d are distributed along the foil…. convention is that clockwise is positive

MORE FORCES

pressure distribution P
units in Pascals
shear stress distribution
units in Pa Force / length^2

Integrate these distributions to obtain the resultant Lift, Drag, and Moments

NORMAL AND AXIAL FORCES

They are with respect to the chord line as opposed to the V_inf free stream

L and D are dimensional
L' and D' are per unit span (like airfoil)

But we need to non-dimensionalize them

Lift has units of N

So we need an N to help

we got **½ rho_inf*v_inf^2*s** (s is chord times span for wings) (this term is dynamic pressure q_inf)

COEFFICIENTS

Cl with lowercase is for a 2D airfoil
CL is L / (q_inf)*s (area)
Cl is L' / (q_inf)*c (chord)

So…..

L = q_inf*s*cL

Similarly
Cd = D’ / q_inf*c

etc etc

There’s also moment

cM = M’/ (q_inf)*c^2   (it’s c^2 ‘cause units)

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cl, cd, cm = f(geometry, alpha, Mach number, Teynold’s number)

Center of Pressure

It is the location where the resultant of the distributed load effectively acts on the body – if moneys are taken about the CP, the integrated effect oof the distributed load would be zero

Pressure Coefficient - Cp

(graph on page 22)

On a graph of Cl vs. alpha, slope is usually 2*pi

For a symmetric airfoil, when alpha = 0, Cl is also zero.

Cl = 2*pi(alpha – alpha0) (valid for the inviscid regime of all airfoil, subsonic)

What about DRAG?

There are three sources of drag:
-Pressure
-Viscous
-Wave

Pressure drag is caused by there being high pressure on the leading and trailing edge and there being low pressure on the top and bottom. There is an adverse pressure gradient when the air flows from high p at the TL to the low p in the middle area

Pressure drag depends on shape

Viscous drag

Occus at the surface
-liquid wants to bring stuff with it

Wave drag coes from schocks and it’s only for transonic and supersonic

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WHAT IS VISCOSITY?

mu us the coefficient of viscosity (dynamic)

mu is in units of Newton-seconds per meter^2

OR pascal-seconds

OR

kg / meter-second

Mu is a function of temperature. At a certain temperature, the mu for air is about 1.983*10^-5 Pa-s

mu_water = 1.00^-3 Pa-s

mu_liquid honey = 10 Pa-s

Remember, viscosity is a property of the fluid, and not the flow!!!!

SIMLARITY PARAMETERS

Reynold’s number (Re)

Re = rho*V*L/mu

density * velocity * Length reference / dynamic viscosity

Re_inf = rho_inf*u_inf*L / mu_inf

OR

you can have different Re at different points

-Mach Number: ratio of flow velocity to speed of sound

M = V / a

Local vecloity/local speed of sound

a = sqrt (gamma * R * T)

VISCOUS DRAG

When fluid is very close to the surface, its velocity is 0

In a very short distance, fluid speeds up to its full potential.
This distance is called the boundary layer.

Sheer stress at a wall

tau_wall = mu**d*v/*d*n for wall

LAMINAR FLOW

Lamina (a thin scale or sheet)

in laminar flow, molecular diffusion is the only mechanism for energy transfer, so it's slow

Turbulent flow is chaotic, irregular, 3d, disorderly and random, unsteady, and you have eddies

Mixing is from eddies

For turbulent flow, the slope bv/bn is high
The air speeds up faster because it's mixing better

tau_wall = mu* bv/bn so
tau_wall, turb is greater than tau_wall laminar

And

skin friction is higher during turbulent flow

Wince tau is tangential to the airfoil, we can define an x_component for drag as tau_x, which is tau_wall*dx

D'_viscous = the integral of tau_lower*dx PLUS tau_upper*dx

You can non-dimensionalize all this by dividing each term by q_inf (dynamic pressure) to get the cf, which is the SKIN FRICTION COEFFICIENT

cf is a function of position

cd, viscous is an INTEGRATED QUANTITITY

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TRANSITION

This is when laminar flow turns to turbulent flow and it's very different from serpartation

something going to speed up the transition:

-rough surface
-more turbulence in the free stream
-adverse pressure gradient
-heating the fluid surface

An adverse pressure gradient means that the pressure increases along the chord length
bP/bx > 0 in other words.

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SEPARATION

(there is a good drawing page 29, bottom)
Separation occurs when bv/bn = 0!!!

Turbulent boundary layers have more energy close to the surface, and therefore it's harder for the boundary layer to separate.

(good pictures, explanation page 30)

STREAMLINES, PATHLINES, STEADY vs. UNSTEADY, ETC

Equation of a streamline is….
(derivation on page 31)

basically it is velocity vector cross with the path vector, and that equals zero

You get dx/u = dy/v = dy/w

PATHLINE

Lie a long exposure shot – tracks particles
you integrate the velocity

dx/dt = u(x,y,z,t) etc

IN A STEADY FLOW
The pathline = the streamline

ANGULAR VELOCITY
-fluids rotate and deform at the same time, so we do perpendicular lines and average out

(The picture with the description and derivation is on page 32)

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(A good pictuire of the Cp chart with pressure distribution is on page 41)

For the upper surface, at x/c is zero, you get the stagnation point, which has the highest pressure.
At the peak of the upper surface, you get the absolute lowest pressure and the highest flow velocity
On the back end yu have an adverse pressure gradient (caused by separateion)

Hgihest Cp = 1 at incompressible flows
if Cp > 1, your flow is compressible!!!

Integrate the cp vs x/c curve to get the coefficient of lift!!

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ANGULAR VECLOICTY

We get another derivation thing on page 42, 43

BASICALLY Angular veclity is rate of change of angular rotaition dtheta1/dt and dtheta2/dt.

dtheta1/dt is equal to the limit as t goes to zero of -bu/by

dtheta2/dt is te limit as t approaches zero of bv/bx

ANGULAR VELOCITY - wz = ½ (dtheta1/dt + dtheta2/dt) (average) = ½ (bv/bx – bu/by)

For 3 dimensions…..

formula on page 43 (it'll look terrible if I write it out here)

What is vorticity?>
Vorticity is defined by TWICE the angular velocity
Denoted by squiggle…it is a vector

The curl of something is the gradient crossed with it (upside-down triangle)

so the curl of velocity is
gradient X veclocity
in 3-dimensions
(b/bxi + b/byj + b/bxk) X (xi + yj + zk)

If the curl is equal to zero, then there is no rotation.
If the curl; is not equal to zero, then the flow is rotational and has rotational veclocity!

RETURNING TO FLUID ELEMENTS

defining the inside angle of the deforming fluid element as kappa, or K
K is euqla to theta1 – theta2
-delta-K is equal to strain that's positive (look at page 45 for details)

"Dilitation" of a fluid is equal to the divergence of the fluid
gradient DOT velocity
which is
bu/bx + bu/by + bw/bz

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REMINDER: ABSOLUTE PRESSURE CANNOT BE NEGATIVE

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KNOWLEDGE OF VISCOUS FLOW IS ESSENTIAL FOR EXTERNAL FLOW APPLCATIONS
-airfoil wing design
-airfoil and wing analysis
-fans, props, helicopters
-wind turbines
-cars, ship and sail shapes

-channels, ducts, and pipes
-transport
-biomedical applications
-heart pumps
flow through arteries, valves

Viscous flow applications – inacting flow
-mizing of species obeys a set of governming laws and equations that are similar to the viscous transport of momentum and energy from one region of the fluid to another

turbulent flow has much more surface area!!

Properties of flow….

viscosity mu
is a property of the fluid no the flow AGAIN
it's derived from intermolecular forces
-normal vs. tangential stress
Viscosity in liquids
As Temperature increases, mu decreases b/c liquid expands, molecules move apart

in GASSES, viscosity is associated with random molecule movement, so as temperature increases, mu increases

FLUIDS

Reminder: strain is infinite, HOOKE'S LAW does not apply
Therefore we should define rate of straing

Newton
-tangental stress is proportional to rate of strain

tau = mu*velocity/height
For his weird plate project

NEWTON'S ESTIMATES OF VISCOSITY

tau = F/A = mu*V/h

This only works for one dimension

STOKES RELATIONS

-Stokes extended Newton's observations to multi-dimensions

-replaced “strain” with “strain rate”
---these relationships can not be proved, only verified in practice

****

STRESS IS A TENSOR

Stress is defined by
-magnitude
-the face on which it acts
-the direction of the force associated with the stress

Temperature is scalar
Velocity is vector
stress is tensor!

(to look at visual examples of this, look on pages 48 and 49)

STOKE’S HYPOTHESIS

lamda = -2/3 * mu

If hydrodynamic pressure conditions are equal to hydrostatic pressure conditions.
Stokes basically made up this relation to mu called lambda

And said that there was conservation of mass control volume
The rate at which mass changes inside a fluid element volume is equal to the rate at which it enters

CONTINUITY IN 2D

brho/bt + brho*u / bx + b(rho*v / by = 0 (page 51)

&**The only assumption is continuum in 2d!!!
In 3d, just add brho*w/bz at the end

in other words

brho/bt + div(rho*v) = 0
AGAIN, NO ASSUMPTIONS EXCEPT CONTINUUM

If we assume incompressible….

We drop the brho/bt term and we can bring out the rho from the div term, and it divides out and we get….

bu/bx + bv/by + bw/bz = 0. This is the continuity equation for incompressible flow. IT'S THE SAME AS DILATION!!!! (Page 52)

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Derivation of the Navier-Stokes equation….

Need to look at teacher's notes!!! It's on pages 53 and 54

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EXACT SOLUTIONS OF NAVIER-STOKES EQUATION
Are rare ☹

There is one situation in which enough of the terms cancel out:
-infinitely long plates
therefore
b/bx = 0 other than pressure term, so bu/bx, bv/bx, bz/bx = 0

The pressure derivative bP/bx is constant, required to dreive the flow

Examples of parallel flow

Either a top plate is moving
OR

There is a pressure differential.

Flow between very long parallel plates and flow through a very long pipe

- ➔ parallel flow – no x derivations
- ➔ so

drho/dt + drho*u/dx + drho*v/dy = 0

If we assume steady flow, then…
There is no variance of density with time so we can get rid of drho/dt

Then, if we assume incompressible, we can take the rho term out and it only becomes du/dx + dv/dy = 0

If we assume parallel flow then we are only left with dv/dy is equal to zero.

THEREFORE, v is f(x) or v is constant
SO v is constant b/c dv/dx is equal to zero

Apply boundary condition at wall
u = v = 0

If the pressure gradient is zero, dP/dx = 0, then this is called Couette flow

if dP/dx is constant this is called a Pouseuille Flow

For dP/dx, look at page 56 and 57 for a derivation and solving of it

you get something like

u = Vy/h + 1/mu * dp/dx * y/2 (y-h)

The first term is the component due to the veclocity of the upper plate

The second term is the component due to the pressure gradient

****

For couette flow…
u(y) = Vy/h

Linear velocicty profile

We can also calculate what the shear stress is at the wall
Txy = mu (du/dy + dv/dx) the dv/dx term goes away, to be Txy = mu* V/h

The coefficient of friction is equal to Twall / (1/2 * rho * V^2)

Or....Cf = 2/Re

PLANE POSEUILLE FLOW

Dp/dx is non-zero, V = 0 -- there is no velocity of the top plate (this isn't through a pipe)

u(y
) = 1/mu * dp/dx * y/2 (y-h) This is a prabolic profile

When $y-h < 0$, there is a favorable pressure gradient
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In a generalized case....

(there is a a nice picture of this on page 58)

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NOW LET'S LOOK AT FLOW THROUGH A PIPE

Continuity $du/dx + dv/dy = 0$

Cylindrical, so
$du/dx + 1/r * d(rv)/dr = 0$

It's infinitely long, so we can assume can assume $d/dx = 0$, except for the pressure gradient term dp/dx

so....

$d(rho*u)/dx + 1/r (d(rho*r*u)/dr = 0$

take out the first term because it's infinitely long....
simplifies to

d(rv)
/dr = 0 ==> rv is constant

v at R = 0
therefore v = 0

****
1. Flow is steady: $d/dt = 0$
2. flow is $d/dtheta = 0$ (not rotating

ux = utheta = 0

$d/dx = 0$

therefore you can use this equation:

$1/r * d/dr * r(du/dr) = 1/mu * dP/dx$

Integrate it twice to get

u(r) = 1/mu * dpdx * r^2/4 + C1lnr + C2

C1 must equal zero

at r = R
left with....
u(r) = -1/4mu * dp/dx[R^2 - r^2]

Which is again, a parabolic profile

dp/dx < 0 creates a favorable pressure gradient
dp/dx > 0 presents an adverse pressure gradient

VOLUME FLOW RATE

Is something that we may run into

It is just the integral from 0 to R of u(r)*2*pi*r*dr

Which is....
-piR^4/8*mu * dp/dx

It's proportional to R^4 not R2, which is weird

---Remember, this is al for laminar flow

Average Veloicty

v = -R^2/8mu * dp/dx

This is volume / Area

SKIN FRICTION

R/2*mu * dp/dx

This is just integrating the shear stress

****
More about Knudsen number
Continuum starts breaking down when Kn is the order of 0.1. If Kn < 0.1, you shouldn't assume continuum. If Kn > 0.1, then you are safe

Kn equals lambda / characteristic length

Some more terms I forgot about:

A.C. (Aircraft Center (maybe?) is when the pitching moment is independent of alpha.

The center of pressure is when the pitching moment is 0. This depends on alpha.

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Slip velocityies in fluids
for Liquids....
slip happens when looking at small scale microflows. Such as water in a 30 micro meter microchannel, observed 10% slip

Gasse
-small knudsen number
-micro/nano flow
-rarefied gasa (low pressures)
-sometimes hypersonic flows
**basically if you have continuum, you are good assuming no-slip

*******EXACT SOLUTIONS FOR NAVIER-STOKES VECLOCITY PROFIELS
(nice picture on page 7

**********************BOUNDARY LAYERS

2D incompressible boundary lyaytr theory
Hydrostatic condition, equivalent to hydrodynamic condition

nu = kinematic viscosity = mu / rho units are m^2/s
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Preliminary remarks about boundary layers
Navier-Stokes equations are 176 years old
-They are non-linear PDE's
-They can't be solved
-superposition of several simple solutions is possible
-as is separation of variables.

Boundary conditions
one on top plate, one on bottom plate for steady flow

Navier-Stokes equations are ELLIPTIC, meaning points in the flow are all coupled to each other. They are computationally expensibve to solve!!!

Navier-Stokes is parabolic PDE, which means it depends on one initial condition

N-S for hypersonic is hyperbolic

****
We need to be able to model the boundary layer
Pradntl (60 years later) decided to try to ignore certain terms
-he did an order of magnitude anakysis.

Prandl divided into 2 regions:

-External region - inviscid assumption
-Boundary Layer - viscous

-characteristic: Very strong gradients: temp velocity

(only relevant for attached oundary layers)

Low alpha still has a boundary layer

Note: a BL is a "shear layer"- a wake is a "free shear layer"
BL rules still apply

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ORDER OF MAGNITUDE ANALYSIS (starts on page 70...it's a derivation)The model consists of a very long surface, L, a freestream in the x directino, V, the width of the boundary layer, delta, and that's it.

So, we get
u = o(V)
x = O(L)
y = O(delta)

Continuity says
dy/dx + dv/dy = 0

O(V/L+ O(v/delta) = 0

They must be the same order@!

So you can say that O(V/L) = O(v/delta))

so..
v = O(Vdelta/ L)
ADN
v/V = O(delta/L)

One of the biggest assumptions is that d <<< L
Therefore, v<<<V

So, we can look at the u momentum equation......

u*du/dq + v du/dy + 1/rho (dp/dx) = mu/rho [d^u/dx^2 + d^2/dy^2)

The LHS simplifies down to
O(VV/L) + O(deltaV.
THIS ORDER OF MAGNITUDE ANALYSIS IS ON PAGE 71